# INDIAN ASSOCIATION OF PHYSICS TEACHERS

## NATIONAL GRADUATE PHYSICS EXAMINATION 2014

**Day & Date of Examination : Sunday, January 19, 2014**

**Time : 10 AM to 1 PM**

Part A- Maximum Marks: 150 Time for Part A: 60 minutes
Part B- Maximum Marks: 150 Timefor Part B : 120 minutes

## Solutions of part A

1. (a) Curl of a curl
$= \nabla\times(\nabla\times E) = \nabla(\nabla.E) - \nabla^2 E \neq 0$

(b) curl of a gradient is

$$\nabla \times (\nabla \phi) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \end{vmatrix} = 0$$

(c) div of a curl is

$$\nabla.(\nabla\times E) = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right).\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = 0$$

(d) grad of a div

$$\nabla(\nabla.E) = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right) \neq 0$$

Ans: b & c

2. The equation of motion can be written as
$m\frac{dv}{dt} = mg - Kv$. On solving we obtain
$v = v_T[1 - e^{-\frac{gt}{Km}}]$

Ans: a

3. In the experiment on Poiseuille's formula the capillary is kept horizontal so as to avoid any effect of gravity and the flow is maintained stream line for small pressure difference. When the pressure difference is made large, the flow velocity exceeds the critical velocity as a result flow becomes turbulent and linearity of the curve is lost.

Ans: b & c

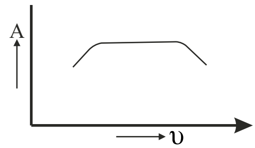
4. Lorentz transformation equations for momentum can be seen in any standard text hence momentum is not invariant under L T.

Ans: b,c & d

5. Clausius equation (Second latent heat equation)
$c_2 = c_1 - \frac{L}{T} + \frac{dL}{dT}$ yields that the specific heat of saturated water vapour is negative. This is so because the saturation is lost on heating. Heat is required to be abstracted from vapour to achieve saturation again.

Ans: c & d

6. The variation of gain with frequency of a class A single stage R-C coupled transistor is shown below.




Ans: c & d

7. Three coplanar forces can be balanced to provide equilibrium

Ans: c

8. The electric field **E** at any point (position vector **r**) inside a uniformly charged (charge density ρ) non conducting sphere is given by $\vec{E} = \frac{\rho\vec{r}}{3\varepsilon_0}$. If there lies a spherical cavity (uncharged inside) and there be a point P inside the cavity at a location $\vec{\mathbf{r}}_1$ with respect to the centre of sphere and $\vec{\mathbf{r}}_2$ with respect to the centre of cavity. Then E at P is obtained by superposition principle as

$$\vec{E} = \frac{\rho\vec{\mathbf{r}}_1}{3\varepsilon_0} - \frac{\rho\vec{\mathbf{r}}_2}{3\varepsilon_0} = \frac{\rho\left(\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2\right)}{3\varepsilon_0} = \frac{\rho\vec{\mathbf{b}}}{3\varepsilon_0}$$

Ans: c

9. One parsec is the unit of distance.
Ans: a

10. In a cubic crystal such as rock salt
$d = \frac{a}{\sqrt{h^2+k^2+l^2}} = 2\overset{\circ}{A}$ . Now using Bragg's law,

$2d \sin\theta = n\lambda$ one obtains
$\sin\theta = \frac{0.710}{2} \Rightarrow \theta = 20.80° = 20°48'$
Ans: c & d

11. Eigen values of an operator having more than one eigen function are said to be degenerate.
Ans: c

12. The solution of the given differential equation with $b = \frac{r}{2m}$ and $\omega = \sqrt{\frac{K}{m}}$ is

(i) Over damped when $b > \omega$
(ii) Critically damped when $b = \omega$
(iii) Damped harmonic when $b < \omega$

Ans: b, & c

13. The process AB and CD in the figure are both isochoric, hence no work is done. BC is an isothermal process hence the internal energy does not change.
Ans: d

14. Unlike a $\pi$ meson & $\mu$ meson, proton is a baryon and has baryon number = 1. The $\pi$ meson is a meson while a $\mu$ meson is a lepton and for each of the meson and the lepton the baryon number is zero.
Ans: b, c & d

15. The extent of acquiring magnetisation is defined as the magnetic susceptibility of a material.
Ans: b

16. The work done is 1 joule along each of the edges AB, BC & CD and zero along other three edges OA, DE & EO.
Ans: c

17. Superposition theorem is "In any linear network containing linear impedances and several sources, the voltage across or the current through any impedance may be calculated by adding algebraically all the individual voltages or currents caused by each source acting alone with all other voltage sources replaced by short circuits and all other current sources replaced by open circuits".
Ans: b, c & d

18. A time varying magnetic field produces an electric field $E = \frac{\pi r^2}{2\pi R}\frac{dB}{dt}$. As a result the torque on the outer ring will be $\tau = qER$.

Thus $\tau = \frac{q\pi r^2}{2\pi R}\beta R = \frac{1}{2}qr^2\beta$
Ans: a

19. Miller indices of the crystal plane are
$h : k : l :: \frac{1}{2} : \frac{1}{3} : \frac{1}{5} ::$ 15 : 10 : 6
Ans: b

20. When a dielectric slab is introduced in an external electric field, all its atoms get polarized. Induced charges $q' = -q(1-\frac{1}{K})$ appear on the surface. The electric field inside the dielectric modifies to the extent $E' = \frac{E}{K}$ . Gauss law applies well under all these situations.

Ans: a, b, c & d

21. According to vector atom model the magnitude $\sqrt{l(l+1)}\,\hbar$ of angular momentum is the eigen value of the angular momentum operator. In the presence of an external magnetic field the angular momentum vector can have $2l+1$ orientations at angles $\cos^{-1}\left(\frac{m}{l(l+1)}\right)$
Ans: c

22. Metallization of an IC means forming of the interconnecting conduction pattern and bonding pads.
Ans: b

23. The nodal slide experiment is based on the properties of nodal points. It employs the principle embodied in option b.
Ans: b

24. Because of their large frequency the carrier waves carry more power while due to small wavelength, the length $\left(l=\frac{\lambda}{4}\right)$ of the antenna is reduced

Ans: b & d

25. Expressions a, b & d are the correct formulae of star delta conversion in electric networks.

Ans: a, b & d

# Part $B_1$

$B_1$ The scalar product of three vectors represents the volume of a parallelopiped whose concurrent edges are represented by three vectors both in magnitude and direction. If the three vectors are coplanar the volume of parallelopiped will be zero because there will be no height then.

$B_2$ The free energy of a thermodynamic system is expressed as $F=U-TS \Rightarrow dF=dU-Tds-SdT$ or $dF = dU - (dU + PdV) - Sdt \Rightarrow dF = -PdV - SdT$. Thus F is unaltered in an isothermal (dT = 0) & isochoric (dV = 0) process.

$B_3$ The intensity distribution in F P Etalon is described by $I=\frac{I_{max}}{1+\frac{4R}{(1-R)^2}\sin^2\left(\frac{\delta}{2}\right)}$ while the sharpness of fringes is dictated by half width of maximum. Putting $I=\frac{1}{2}I_{max}$ half power points are obtained as

$$\frac{I}{I_{max}}=\frac{1}{2}=\frac{1}{1+\frac{4R}{(1-R)^2}\sin^2\left(\frac{\delta}{2}\right)}$$

$$\Rightarrow \frac{4R}{(1-R)^2}\sin^2\left(\frac{\delta}{2}\right)=1 \Rightarrow \delta = 2\sin^{-1}\left(\frac{(1-R)}{2\sqrt{R}}\right)$$

Using the value of reflection coefficient R = 0.8, the half width is $\delta$=0.22 rad. Intensity distribution in case of Michelson interferometer is expressed as $I=I_{max}\cos^2\frac{\delta}{2}$. Putting $I=\frac{1}{2}I_{max}$ at half power points, the half angular width of maximum is obtained as $\delta = 2\cos^{-1}\frac{1}{\sqrt{2}}$

$\Rightarrow \delta = \frac{\pi}{2} = 1.57$ which is 7 times the half width of the maximum of F P fringes.

$B_4$ Heisenberg uncertainty principle gives $\Delta E \times \Delta t \cong \frac{\hbar}{2}$ According to Bohr quantum condition the electron revolves in definite quantized energy states where the angular momentum is $mvr = n\hbar$. Each of these states are considered to have a sharply defined energy E, such that $\Delta E = 0 \Rightarrow \Delta t = \infty$ which means that the energy states have infinite life time but the excited energy states are found to have a life time $\Delta t = 10^{-8}\,s$. Thus the concept of definite Bohr orbits clearly violates uncertainty principle. This finite life time of an energy state yields finite width (uncertainty) of energy levels $E=\frac{\hbar}{2\times10^{-8}} \cong 10^{-8}\,eV$ which predicts that the radiations emitted when atom de excites to ground state from a particular higher state, are not truly mono - chromatic. In other words the spectral lines can never be infinitely sharp as demanded by Bohr theory, rather have a finite width hence the statement is refuted.

$B_5$ The given wave function is $\psi(x,t)=x^2+c^2t^2$ Differentiating with respect to space and time respectively, it yields $\frac{\partial^2\psi}{\partial t^2}=c^2\frac{\partial^2\psi}{\partial x^2}$ which is the differential equtation of a wave travelling with velocity = c.

$B_6$ The saturation current in LR circuit, just before turning the switch from B to C, is $i_0=\frac{E}{R}$ While the time constant is $\tau=\frac{L}{R}$.

The magnetic energy stored during the growth of current is $=\frac{1}{2}Li^2=\frac{1}{2}L\frac{E^2}{R^2}$

Once the switch in the circuit is turned from B to

C, the cell is no longer in the circuit and the energy stored in the inductor (L) starts dissipating. The current (i) as a function of time is expressed as $i = \frac{E}{R} e^{-Rt/L}$. The total energy dissipated in the resistance R after turning the switch from B to C is.

$$\int_0^\infty i^2 R dt = \frac{E^2}{R} \int_0^\infty e^{-\frac{2Rt}{L}} dt = -\frac{E^2}{R} \times \frac{L}{2R} \left\{ e^{-\frac{2Rt}{L}} \right\}_0^\infty$$

$= \frac{E^2 L}{2R^2}$ Thus the total energy dissipated during decay is equal to the energy stored $\frac{1}{2} Li^2 = \frac{E^2 L}{2R^2}$ during growth of current in L-R circuit.

$B_7$ The structure of an atom is such that every nucleus is surrounded by an electrostatic potential barrier that opposes the entry and the escape of positive particles. It is therefore concluded that a compound nucleus formed by absorbing a neutron is most likely to decay by emitting γ-rays which carry no charge rather than a proton, deuteron or α-particle which are all positive.

$B_8$ For monochromatic travelling wave $\upsilon = \frac{c}{\lambda}$ *or* $d\upsilon = -\frac{c}{\lambda^2} d\lambda$ now using coherent length $l_c = \frac{\lambda^2}{\Delta\lambda}$ *the magnitude of* $\Delta\upsilon = |d\upsilon|$ *can be written as* $\Delta\upsilon = \frac{c}{l_c}$ *further using* $l_c = c\tau_c$ we get $\Delta\upsilon = \frac{1}{\tau_c}$. Therefore the frequency spread of a spectral line is of the order of the inverse of coherence time $\tau_c$ showing that the perfectly monochromatic spectral line with $\Delta\upsilon = 0$ means having infinite coherence time may not be possible. This is why the concept of temporal coherence is intimately connected with mono-chromaticity. Further the quantity $Q = \frac{\lambda}{d\lambda}$ is taken to represent the mono-chromaticity or the spectral purity of the source. Thus one obtains $l_c = \frac{\lambda^2}{\Delta\lambda} = Q\lambda$.

$B_9$ The atoms which have only one electron outside the completely filled (closed) shells are known as one electron atom Hydrogen (Z =1) and alkali atoms Li (Z=3), Na(Z=11) K(Z=19), Rb(Z=37) and Cs(Z=55) belong to this class. The closed shells in these atoms do not contribute to orbital and spin angular momentum. It is only the valence electron whose states characterize the optical spectra of such atoms. For alkali atoms the lowest energy state is n > 1 rather than n = 1 as in the case of hydrogen. The spectra of alkali atoms consist of the following series with specified transitions.

1. Sharp Series: s-state to p-state
2. Principal Series: p-state to s-state
3. Diffuse Series: d-state to p-state
4. Fundamental Series: f-state to d-state

The spectra of diatomic molecules such as $H_2$ molecule are majorly

a. Rotational spectra
b. Rotational-viberational spectra
c. Electronic-viberational spectra
d. Electronic-rotational spectra

$B_{10}$ The attractive magnetic and repulsive electric forces between two like charges running parallel can be expressed as

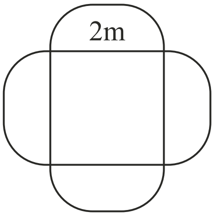
$$F_M = \frac{\mu_0}{4\pi} \frac{qv \times qv}{R^2} \text{ and } F_E = \frac{1}{4\pi\varepsilon_0} \frac{q \times q}{R^2}$$

Thereby

$$\frac{F_M}{F_E} = \mu_0 \varepsilon_0 v^2 = \frac{v^2}{c^2}$$

# Part $B_2$

P1. The moment of inertia, about an axis perpendicular to its plane and passing through its centre is $I_s = \frac{ml^2}{6}$ of a square lamina and $I_c = \frac{mR^2}{2}$ for a circular lamina.




Let $M_s$ and $M_c$ be the masses of the square and each semicircular lamina in question. So that $M_s + 4M_c = M = 5$ kg ---- (1). If $\sigma$ be the surface density, then

$$M = \sigma l^2 + 4\sigma \frac{\pi R^2}{2} = \sigma l^2\left(1+\frac{\pi}{2}\right) \Rightarrow \sigma = \frac{7M}{72}$$

Thereby $M_s = \frac{7M}{18}$ & $M_c = \frac{11M}{72}$ ............(2)

Further since the centre of mass of uniform semicircular lamina lies at $4R/3\pi$ from centre, therefore the moment of Inertia about an axis perpendicular to its plane and passing through its centre of mass is obtained by using the theorem of parallel axis as $I_0 = \frac{M_c R^2}{2} - M_c\left(\frac{4R}{3\pi}\right)^2$ Further using theorem of parallel axis again, the required moment of inertia of one semicircular section about the given axis is $I_c = \left[I_0 + M_c\left(\frac{l}{2}+\frac{4R}{3\pi}\right)^2\right]$

substituting $I_0$

$$I_c = \left[\frac{M_c R^2}{2} - M_c\left(\frac{4R}{3\pi}\right)^2 + M_c\left(\frac{l}{2}+\frac{4R}{3\pi}\right)^2\right] \quad \text{or}$$

$I_c = M_c\left[\frac{R^2}{2}+\frac{l^2}{4}+\frac{4lR}{3\pi}\right]$ Finally $I = I_s + 4I_c$

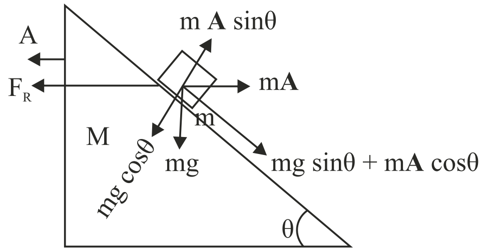
$$I = \frac{M_s l^2}{6} + 4M_c\left[\frac{R^2}{2}+\frac{l^2}{4}+\frac{4lR}{3\pi}\right] \text{ kgm}^2$$

substituting the values of $M_s$ and $M_c$

$$I = \frac{7Ml^2}{18\times 6} + \frac{4\times 11M}{72}\left[\frac{R^2}{2}+\frac{l^2}{4}+\frac{4lR}{3\pi}\right] \text{ using } l = 2R$$

$$I = 1.69\frac{Ml^2}{4} = 1.69MR^2 = 8.45\, kgm^2$$

P2. When left free the small block of mass m slides down with acceleration **A** and in c and in turn the wedge of mass M moves left with an acceleration **A**.




The two equations of motion can be written as
mg sinθ + mA cosθ $= m$ a -- (1)

$$F_R = [mg\cos\theta - m\mathbf{A}\sin\theta]\cos(90-\theta) = M\mathbf{A} \quad (2)$$

$$mg\sin\theta\cos\theta = (M + m\sin^2\theta)A$$

$$\mathbf{A} = \frac{g\sin\theta\cos\theta}{\frac{M}{m}+\sin^2\theta}$$

Differentiating with respect to θ

$$mg(\cos^2\theta - \sin^2\theta)$$

$$= (M+m\sin^2\theta)\frac{dA}{d\theta} + 2mA\sin\theta\cos\theta$$

For maximum A, setting $\frac{dA}{d\theta} = 0$

we get $g(\cos^2\theta - \sin^2\theta) = 2A\sin\theta\cos\theta$

$$g(\cos^2\theta - \sin^2\theta) = 2\frac{mg\sin\theta\cos\theta}{M+m\sin^2\theta}\sin\theta\cos\theta$$

$$(1-2\sin^2\theta)(M+m\sin^2\theta) = 2m\sin^2\theta\cos^2\theta$$

$$(1-2\sin^2\theta)(M+m\sin^2\theta)$$
$$= m\sin^2\theta + m\sin^2\theta(1-2\sin^2\theta)$$

$$\left(1-2\sin^2\theta\right)M = m\sin^2\theta \Rightarrow \sin^2\theta = \frac{M}{(m+2M)}$$

$$\Rightarrow \theta = \sin^{-1}\sqrt{\frac{M}{(m+2M)}}$$

P3. (a) The intensity level of sound is $IL = 10\log\frac{I}{I_0} = 105(given)$ using $I_0 = 10^{-12}\frac{\mathrm{W}}{\mathrm{m}^2}$

$I = I_0(10)^{\frac{105}{10}} = \frac{P}{2\pi R^2}$ by definition, then

$$P = 2\pi\,(20)^2 \times 10^{-12}\,(10)^{\frac{105}{10}} = 80\,W$$

(b) The intensity of sound is $I = \frac{p^2}{2\rho v}$

The velocity of sound at T = 293 K is expressed as $v_{293} = v_0\sqrt{\frac{293}{273}} = 332\sqrt{\frac{293}{273}}$

using $\rho = 1.20\,\mathrm{kg/m^3}$ and $v_{273} = 332\,\mathrm{m/s}$

$$I = \frac{\left(6\times 10^{-5}\right)^2}{2\times 1.2\times 332}\sqrt{\frac{273}{293}} = 4.36\times 10^{-12}\,\frac{W}{m^2}$$

Now the intensity level is $IL = 10\log\frac{I}{I_0}$

Further using $I_0 = 10^{-12}\ \mathrm{W/m^2}$ we get

$$\mathrm{IL} = 10\log\frac{6.36\times 10^{-12}}{10^{-12}} = 8\,db$$

P4. At any temperature T, the number of molecules in a given range of speeds between v and v+dv is given by Maxwell Boltzmann distribution law according to which the probability of a molecule to have energy E is proportion to $e^{-\frac{E}{kT}}$ where k is Boltzmann constant. For an ideal gas the energy of molecules is purely kinetic energy hence $\mathrm{E} = \frac{1}{2}\mathrm{mv}^2$. Thus the number of molecules having speed between v and v+dv in a given six dimensional vol $d\Gamma = dxdydzdv_x dv_y dv_z$ is $dn = Ce^{-\frac{E}{kT}}d\Gamma = Ce^{-\frac{mv^2}{2kT}}\,dxdydzdv_x dv_y dv_z$

Now considering a sphere of radius v then infinitesimal volume between radii v and v+dv can be written as $d\Gamma = V\,4\pi v^2\,dv$. The total number of particles will then be

$$N = CV\int_0^\infty e^{-\frac{mv^2}{2kT}}\,4\pi v^2 dv = CV\frac{4\pi}{\left(\frac{4m}{2kT}\right)}\sqrt{\frac{\pi}{\frac{m}{2kT}}}$$

$$N = CV\left(\frac{2\pi kT}{m}\right)^{\frac{3}{2}} \Rightarrow CV = N\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}$$

Substituting the value of C we get $dn = 4\pi N\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}}dv$ If P(v) represent the the probability that a molecule will have velocity between v and v+dv in the velocity range zero to infinity then

$$P(v) = 4\pi N\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}}$$

The most probable velocity is the value of v for which the function P(v) is maximum. Therefore

$$\frac{dP(v)}{dv} = 2ve^{-\frac{mv^2}{2kT}} + v^2 e^{-\frac{mv^2}{2kT}}\left(-\frac{mv}{kT}\right) = 0$$

or $2 + v\left(-\frac{mv}{kT}\right) = 0 \Rightarrow v_{mp} = \sqrt{\frac{2kT}{m}}$

Also the root mean square velocity is

$$v_{rms} = \sqrt{\frac{\int_{-\infty}^{+\infty} v^2 dn}{\int_{-\infty}^{+\infty} dn}} = \sqrt{\frac{\int_{-\infty}^{+\infty} v^4 e^{-\frac{mv^2}{2kT}}dv}{\int_{-\infty}^{+\infty} v^2 e^{-\frac{mv^2}{2kT}}dv}}$$

$v_{rms} = \sqrt{\frac{3kT}{m}} \Rightarrow \frac{v_{rms}}{v_{mp}} = \sqrt{\frac{3}{2}}$ Hence proved

P5. The light reflected strongly back from the glass plate is really the source of glare. A thin film which can produce destructive interference in reflected light shall reduce the glare and the sculpture will be seen clearly by refraction. This can be achieved by coating the glass ($\mu$=1.62) surface by a thin film of either $MgF_2$ ($\mu$=1.38) or diamond ($\mu$=2.42). The conditions being different in the two cases. The phase difference caused by a thin transparent film between two light beams, reflected successively from the two surfaces of a film is

$\phi = \frac{2\pi}{\lambda}\Delta + \pi$ (due to Stokes' law)

$\phi = \frac{2\pi}{\lambda}2\mu t\cos(\alpha + r) + \pi$ (due to stoke's law)

For destructive interference in reflected light (in case of thin diamond film $\mu$ = 2.42)

$$\frac{2\pi}{\lambda}2\mu t\cos(\alpha+r)+\pi=(2n+1)\pi$$

For a thin parallel film the wedge angle α =0 and for normal incidence r = 0 implies that

$$2\mu t=n\lambda\Rightarrow t=n\frac{\lambda}{2\mu}=1214,\ 2428,\ 3642\ A^\circ$$

On the other hand, there shall not be stoke's contribution to the phase difference in case of $MgF_2$ (μ=1.38) film as the phase change of π occur on each face of the film, therefore

$$\phi=\frac{2\pi}{\lambda}\Delta \text{ or } \phi=\frac{2\pi}{\lambda}2\mu t\cos(\alpha+r)=(2n+1)\pi$$

Again taking cos($\alpha$ + r) = 1 for a parallel film under normal incidence, therefore

$$t=(2n+1)\frac{\lambda}{2\times2\mu}=1065,\ 3193, 5322\,A^\circ$$

P6. In the given solenoid of rectangular cross section, the current i through its winding produces a magnetic field. Let B be the magnetic field produced at a radius r. Ampere's law in electromagnetism provides

$$\oint B.dl=\mu_0\mu_r\sum i \text{ or } B.2\pi r=\mu_0\mu_r Ni$$

$\Rightarrow B=\mu_0\mu_r\dfrac{Ni}{2\pi r}$ weber/m$^2$ where N is the total number of winding over a length 2πr. Considering small area h × dr, the flux.

$$\Phi=N\int_a^b Bh\times dr=\int_a^b\mu_0\mu_r\frac{N\overset{2}{i}}{2\pi r}hdr=\frac{\mu_0\mu_r N\overset{2}{i}h}{2\pi}\ln\frac{b}{a}\text{ W}$$

Using now $\Phi=NLi\Rightarrow L=\dfrac{\mu_0\mu_r N^2h}{2\pi}\ln\dfrac{b}{a}$

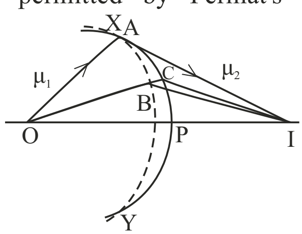
With the given values

$$L=\frac{\mu_0}{4\pi}\times2\times1240\times(1000)^2\times0.01\ \ln\frac{10}{5}$$

$L$= 2.48 ln2 = 1.72 h & total flux linked is

$$\Phi=Li\Rightarrow\Phi=1.72\times10\times17.2\ =\text{Wb}$$

P7. (a) Let the path OAI be the actual path of light ray permitted by Fermat's principle for



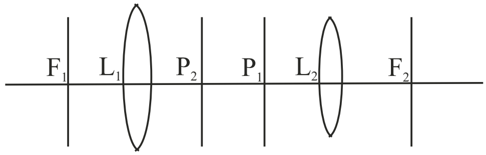

refraction of light through a concave surface with $\mu_2>\mu_1$ means ray of light goes from a rarer medium to a denser medium. Let the curve XY represents the locus of $\mu_1 OA+\mu_2 AI=const$ which means

$$\mu_1 OAB+\mu_2 AI=\mu_1 OB+\mu_2 BI$$
$$=\mu_1(OB+BC)+\mu_2 BI-\mu_1 BC-\mu_2 CI+\mu_2 CI$$
$$<\mu_1(OC)+\mu_2 CI-\mu_2(BC+CI-BI)$$

Further since BC+CI–BI>0 always, one can say that in any case (for all paths) $\mu_1 OA+\mu_2 AI<\mu_1 OD+\mu_2 DI$. The actual path is therefore a minimum i,e extremum.

(b) Huygens' eyepiece consists of two plano-convex lenses of focal lengths $f_1$ = 3$f$ (field lens) and $f_2=f$ (eye lens) mounted co-axially in a metallic tube at a separation of $d=2f$. The focal length of the eyepiece as a whole is

$$F=-\frac{f_1f_2}{d-(f_1+f_2)}$$




The two principal points $P_1$ and $P_2$ of the eye piece are located as $L_1P_1=+\dfrac{Fd}{f_2}$ i,e on the right of field lens and $L_2P_2=-\dfrac{Fd}{f_1}$ i,e on the left of eye lens. The two focal points $F_1$ and $F_2$ are located such as $P_1F_1=-F$ and $P_2F_2=+F$ As long as the media on the two sides of the eyepiece are the same, the two nodal points coincide with the respective principal points. Since it is difficult to use a crosswire in the first focal, plane of Huygen's eye piece, Ramesden's eyepiece is preferred in all instruments used for physical measurements.

P8. (a) According to Fermi Dirac distribution the probability occupancy is $f(\text{E})=\dfrac{1}{e^{\frac{E-E_F}{kT}}+1}$. rather than $f(\text{E})=\dfrac{1}{e^{\frac{E-E_F}{kT}}-1}$. Let us consider two energy levels $E_1$ and $E_2$ which are equally spaced above and below the Fermi level such that $E_1=E_F+\Delta E\ and\ E_2=E_F-\Delta E.$ Now the probability of occupancy of the level $E_1$ is

$$f(E_1) = f(E_F + \Delta E) = \frac{1}{e^{\frac{E_F + \Delta E - E_F}{kT}} + 1} \qquad (1)$$

The probability of occupancy of the level $E_2$ is

$$f(E_2) = f(E_F - \Delta E) = \frac{1}{e^{\frac{E_F - \Delta E - E_F}{kT}} + 1} \qquad (2)$$

The total probability is

$$f(E_1) + f(E_2) = \frac{1}{e^{\frac{+\Delta E}{kT}} + 1} + \frac{1}{e^{\frac{-\Delta E}{kT}} + 1} \quad \text{or}$$

$$f(E_1) + f(E_2) = \frac{1}{e^{\frac{+\Delta E}{kT}} + 1} + \frac{e^{\frac{+\Delta E}{kT}}}{1 + e^{\frac{+\Delta E}{kT}}} = 1$$

Hence proved.

(b) The electron energy in silver is $E = E_F + \frac{E_F}{100} = \frac{101}{100} E_F$. Now the occupancy of this state is

$$f(E) = \frac{1}{e^{\frac{E - E_F}{kT}} + 1} = \frac{1}{e^{\frac{\frac{101}{100}E_F - E_F}{kT}} + 1} = \frac{1}{e^{\frac{E_F}{100kT}} + 1}$$

Now

$$\frac{10}{100} = \frac{1}{e^{\frac{E_F}{100\,kT}} + 1} \Rightarrow e^{\frac{E_F}{100\,kT}} + 1 = 10$$

$$\Rightarrow \frac{E_F}{100kT} = \ln 9 \Rightarrow T = \frac{E_F}{100k \ln 9}$$

$$\Rightarrow T = \frac{5.5 \times 1.6 \times 10^{-19}}{100 \times 1.38 \times 10^{-23} \ \ln 9} = 290 \text{ K}$$

P9. The commutator $\left(\frac{d}{dx}, V\right)$ can be expanded as

$$\left(\frac{d}{dx}, V\right)\Psi = \frac{d}{dx}(V\Psi) - V\frac{d}{dx}(\Psi)$$

$$\text{or } \left(\frac{d}{dx}, V\right)\Psi = \frac{dV}{dx}\Psi + V\frac{d\Psi}{dx} - V\frac{d\Psi}{dx} = \frac{dV}{dx}\Psi$$

$$\Rightarrow \left(\frac{d}{dx}, V\right)\Psi = \frac{dV}{dx}\Psi \text{ or } \left(\frac{d}{dx}, V\right) = \frac{dV}{dx}$$

hence proved.

Further if the eigen function of a quantum system be $\Psi(x,t) = Ae^{-i(\omega t - kx)}$.

Then $\frac{\partial \Psi}{\partial t} = -i\omega A e^{-i(\omega t - kx)}$ which implies

$$i\hbar\frac{\partial \Psi}{\partial t} = -i^2\hbar\omega\Psi(x,t) = \hbar\omega\Psi(x,t) = E\Psi(x,t)$$

and $\frac{\partial^2 \Psi}{\partial x^2} = (ik)^2 A e^{-i(\omega t - kx)}$ now multiplying by $-\hbar^2$ we get

$$\Rightarrow -\hbar^2\frac{\partial^2 \Psi}{\partial x^2} = -\hbar^2 (ik)^2 \Psi(x,t) = \hbar^2 k^2 \Psi(x,t)$$

Also we can write $E = \frac{p^2}{2m} + V$ or

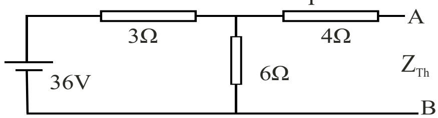
$$\hbar\omega = \frac{(\hbar k)^2}{2m} + V \Rightarrow \hbar\omega\Psi = \left(\frac{(\hbar k)^2}{2m} + V\right)\Psi$$

which in turn can be written as

$$i\hbar\frac{\partial \Psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\right]\Psi \Rightarrow i\hbar\frac{\partial \Psi}{\partial t} = H\Psi$$

This is well known Schrodinger Equation.

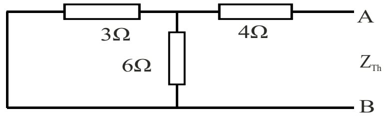
P10. In order to draw Thevenin equivalent circuit,




we consider the open circuit voltage as $V_{th} = V_A$ $V_B$ with no load impedance between A & B. Then the nodal analysis gives

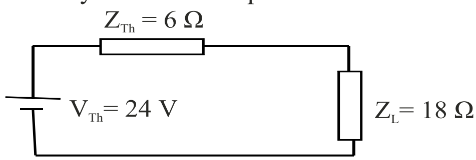
$$\frac{V_{th} - 36}{3} + \frac{V_{th} - 0}{6} = 0 \Rightarrow V_{th} = 24\ V$$

To find Thevenin impedance across A & B, we consider voltage sources short circuited and current sources open circuited as below




then $Z_{Th} = 4 + (3 \parallel 6) = 4 + \frac{3 \times 6}{3 + 6} = 6\ \Omega$

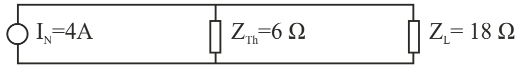
Thereby the Thevenin equivalent circuit is as below




Thevenin current will therefore be

$$I_{Z_L} = \frac{V_{Th}}{Z_{Th} + Z_L} = \frac{24}{6 + 18} = 1\ A$$

Using the concept of duality between voltage and current source, Norton equivalent circuit is as drawn below,




$I_N = \frac{V_{Th}}{Z_{Th}} = \frac{24}{6} = 4A$ further applying KCL,

hence the $\frac{V - 0}{6} + \frac{V - 0}{18} = 4 \Rightarrow V = 18\ V$

current through $Z_L$ is $I_{Z_L} = \frac{V}{Z_L} = \frac{18}{18} = 1A$