INDIAN ASSOCIATION OF PHYSICS TEACHERS

NATIONAL GRADUATE PHYSICS EXAMINATION 2011

Date of Examination : Sunday 23rd January 2011

Time : 10 AM to 1 PM

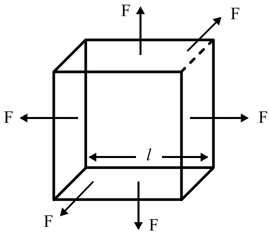
Solutions of part A

1 Curl of a conservative force is zero. The curl $\nabla \times (x^2 i + y^2 j + z^2 k) = 0$

curl of a gradient is zero. $\therefore \nabla \times (\nabla U) = 0$

Ans **a, b**

2 Let the given cube is stretched out by a force F in all x,y and z directions.



The strain is $\left(\frac{\Delta l}{l}\right)_{longitudenal} = \frac{F}{AY} \Rightarrow \Delta l = \frac{F}{AY}$

Because A=1 m^2 & l = 1m. Further the lateral strain is $\left(\frac{\Delta l}{l}\right)_{lateral} = \frac{\sigma F}{AY} \Rightarrow \Delta l = \frac{\sigma F}{AY}$

Therefore the new edge of the cube is

$$1 + \frac{F}{Y} - \frac{\sigma F}{Y} - \frac{\sigma F}{Y} = 1 + \frac{F}{Y}(1 - 2\sigma)$$

The new volume becomes (as Y is too large)

$$V = \left[1 + \frac{F}{Y}(1 - 2\sigma)\right]^3 = 1 + 3\frac{F}{Y}(1 - 2\sigma)$$

Volume strain $\frac{\Delta V}{V} = \frac{3F}{Y}(1 - 2\sigma)$

Ans **a, c**

3 Interference is the physical phenomenon of redistribution of energy. Therefore energy remains conserved when two waves interfere.

Ans **c**

4 The excess pressure inside a bubble is $\Delta p = \frac{4T}{R}$

When the bubble is uniformly charged, the mechanical force per unit surface is

$$\frac{\sigma^2}{2\varepsilon_0} = \frac{1}{2\varepsilon_0}\left(\frac{q}{4\pi R^2}\right)^2 = \frac{q^2}{32\pi^2\varepsilon_0 R^4}\ \text{N/m}^2$$

As a result of superimposition of the two, the excess pressure turns out to be

$$\Delta p = \frac{4T}{R} - \frac{q^2}{32\pi^2\varepsilon_0 R^4}\ \text{N/m}^2$$

Ans **d**

5 The torque acting on a current carrying coil is $\tau = NiA \times \mathbf{B} = \mathbf{p_m} \times \mathbf{B}$ The torque is off course, maximum when $\mathbf{p_m} \perp \mathbf{B}$ i,e **B** is in the plane of the coil. Further the torque $\tau = Ni\pi\left(\frac{1}{2\pi N}\right)^2 \mathbf{B}$ is maximum when N is least i,e just N = 1 and this maximum torque is $\tau_{max} = i\pi\left(\frac{1}{2\pi}\right)^2 \mathbf{B}$

$$= 4.51 \times 10^{-3}\pi\left(\frac{0.25}{2\pi}\right)^2 \times 5.71 \times 10^{-3} = 1.28 \times 10^{-7}\text{Nm}$$

Ans **a, c, d**

6 Kepler's secomd law states that the planets revolve round the sun in such a way that the radius vector sweeps equal area in equal interval of time which is a consequence of conservation of angular momentum.

Ans **d**

7 Huygens eyepiece consists of two plano convex lens of focal length 3f and f separated by a distance d=2f. The focal length of the eyepiece turns out to be F=1.5f=3cm (given) So d = 4 cm.

Ans **d**

8 The efficiency of Carnot engine is

$$\eta = \frac{W}{Q_1} = \frac{T_1 - T_2}{T_1} = \frac{900-300}{900} = \frac{2}{3} = 0.667$$

$\eta = 0.667 \times 100\% = 66.7\%$

$W = \eta Q = 0.667 \times 1000 \times 1000 \times 4.2J = 2.8MJ$

Also $W = \frac{2.8 \times 10^6}{60 \times 60}$ Wh $= \frac{2800}{3600}$ kWh = 0.78kWh

Also $W = \frac{2.8 \times 10^6}{10^6 \times 1.6 \times 10^{-19}} = 1.7 \times 10^{19}$MeV

Ans **a, b, c, d**

9 An optical communication fiber is superior to a copper cable as it is light weight and small in size. Further it is very difficult to tap into a fiber cable to steal the data hence it maintains a high level of privacy and security. By using multiplexers, one fiber could replace hundreds of copper cables. The real benefits in the data industry are its immunity to Electromagnetic Interference including the radio frequency interference. Further the fact that the glass is not a conductor makes it possible that it can be used where electrical isolation is needed with no fear of electromagnetic induction. It has a wider frequency band width of the order of tera herz. Fibers pose no threat in dangerous environments such as chemical plant where a spark could trigger an explosion. It seems to be less hazardous as no E M field is produced around the fiber.

Ans **a, b, c, d**

10 See standard text: Hand book of Nanophysics by John D Millar.

Ans **a, b, c**

11. Unlike the conventional acoustic waves, the velocity of ultrasonics varies with frequency. One of the methods of their production is based on piezo-electric effect. These are the mechanical waves of frequency greater than 20 kHz.

Ans **b, c,**

12 The large metallic sheet (conducting sheet) provides an electrical image of the charged bob at a distance h behind it. Thereby the tension in the string will be $T = mg + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2h)^2}$

Ans **c**

13 See standard text Quantum Mechanics by L. Schiff.

Ans **c**

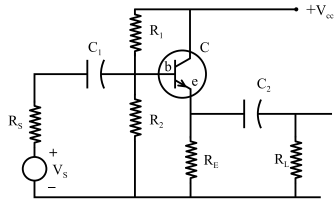
14 The condition of diffraction for a grating is $(e+d)\sin\theta = n\lambda$ Differentiating w r t λ,

$(e+d)\cos\theta \frac{d\theta}{d\lambda} = n$. The dispersive power $\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}$ is therefore the same for both.

While the Resolving power $\frac{\lambda}{d\lambda} = nN$ is not the same for the two.

Ans **b, c**

15 Emitter follower is a negative feed back amplifier circuit with gain less than 1. The circuit is said to be a common collector circuit because the collector is common to both the input and the output and is at ac ground. In such a circuit both the output and input are in phase.



Ans **b, c, d**

16 In a canonical ensemble the density of states is given by $\rho \propto e^{-\frac{\varepsilon}{kT}}$ and the macro state of the system is described by the variables (TNV)

Ans **b**

17 Magnetic susceptibility (χ) is defined as $\chi = \frac{I}{H} = \frac{p_m}{VH}$ Also Curie law states that

$$\chi = \frac{\text{constt}}{T} \quad \text{or} \quad \frac{p_m}{VH} = \frac{\text{constt}}{T}$$

$$\Rightarrow \frac{\{p_m\}_{|at\,4\,K\,\&\,1T}}{\{p_m\}_{|at\,3\,K\,\&\,1.5T}} = \frac{H_1}{T_1} \times \frac{T_2}{H_2} = \frac{1 \times 3}{4 \times 1.5} = \frac{1}{2}$$

Thus the magnetic moment $(P_m) = 10.0\ Am^2$

Ans **c**

18 In an ohmic conductor such as copper the current density is $J = nev_d = ne\mu\mathbf{E}$. Also $J = \sigma E$

$$\Rightarrow \sigma = ne\mu \quad \text{or} \quad \mu = \frac{\sigma}{ne} = \frac{6 \times 10^7}{9 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$\mu = 4.16 \times 10^{-3} \frac{m^2}{VS} = 4.16 \times 10^{-3} \frac{m^2}{\Omega C}$$

Ans **b,d**

19 The energy eigen values of a particle in a one dimensional potential well are given by $E_n = \frac{n^2\hbar^2}{2ma^2}$

Ans **a,c**

20 Maxwell's equations are applied to all kinds of medium i,e conducting, dielectric, plasama and even for vacuum. In each case the boundary conditions may be different.

Ans **a,b,c,d**

21 Regular reoccurrence of peaks in the binding energy curve for certain nuclei denotes extra stability of these nuclei. This indicates that the nuclei in which the nucleon number is a multiple of 4 are most stable. Four nucleons forms a group of two proton and two neutrons which is just an α-particle hence an α-particle model.

Ans **b**

22 According to Langevin, the number and orientation of electronic orbits in an atom/molecule of a paramagnetic substance is such that the resultant magnetic dipole moment of the atom, as a whole, is non zero and the interaction between these molecular dipoles is considered to be negligible.

Ans **b**

23 Fermi Dirac distribution is represented by the curve 1 at a non zero temperature while by curve 2 at absolute zero.

Ans. **a, b**

24 The given specifications are for a Tetragonal lattice $a = b \neq c$ & $\alpha = \beta = \gamma = 90^0$

Ans **b**

25 The Madelung constant is the representation of cohesive energy of ionic crystals. In case of NaCl any ion Na or Cl has 6 nearest neighbors of opposite charge at a distance a and 12 next neighbours at a√2 and then 8 at a√3 and so on, resulting into the total interaction energy as

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Ne^2}{a}\left\{\frac{6}{\sqrt{1}} - \frac{12}{\sqrt{2}} + \frac{8}{\sqrt{3}} - \frac{6}{\sqrt{4}} + \ldots\right\}$$

$$= \frac{ANe^2}{4\pi\varepsilon_0 a} = 1.7476\,\frac{Ne^2}{4\pi\varepsilon_0 a}$$

Here A= 1.7476 is Madelung constant

Ans **a, c**

Part B -I

B1 For a vector of constant magnitude $\mathbf{R}.\mathbf{R} = |R|^2 = C$

Differentiating w r t time (t). $\mathbf{R}.\frac{d\mathbf{R}}{dt} + \frac{d\mathbf{R}}{dt}.\mathbf{R} = 0$

$\Rightarrow \mathbf{R}.\frac{d\mathbf{R}}{dt} = 0 \Rightarrow \frac{d\mathbf{R}}{dt}$ is $\perp$ to R when $\frac{d\mathbf{R}}{dt} \neq 0$

Thus showed.

B2 Normally the specific heat of gases is thought to be independent of temperature but experiments have shown its temperature dependence. In general the total energy of one gram-mole of a diatomic gas is expressed as

$$E = E_{translational} + E_{rotational} + E_{vibrational}$$

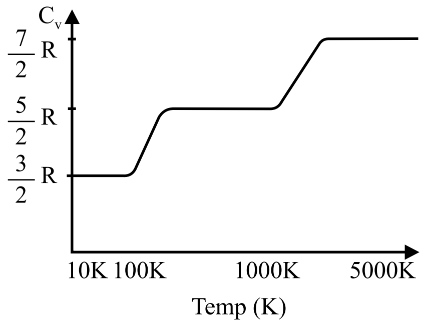
Let the corresponding specific heat be $C_v = C_t + C_r + C_\upsilon$. A diatomic gas can have 3 degrees of freedom for translation, 2 for rotation and 2 for vibrational motion. The contribution to specific heat is $\frac{R}{2}$ per degree of freedom.

Therefore $C_v = 3 \times \frac{R}{2} + 2 \times \frac{R}{2} + 2 \times \frac{R}{2}$

At very low temperature i.e. for $T < \frac{h}{2\pi\ Ic}$ only the translation motion is excited and not rotation and vibration therefore the specific heat is, $C_v = \frac{3}{2}R$. With increase in temperature the rotational motion is also excited and then the specific heat is $C_v = 3 \times \frac{R}{2} + 2 \times \frac{R}{2} = \frac{5}{2}R$ as long as $T < \frac{h}{R}\sqrt{\frac{K}{\mu}}$ where symbols have usual meaning. At high temperature the vibration motion is also excited and then the specific heat turns out to be

$$C_v = 3 \times \frac{R}{2} + 2 \times \frac{R}{2} + 2 \times \frac{R}{2} = \frac{7}{2}R$$

The variation of specific heat over the given range is depicted here.



B3 "Whether the conduction current through a specimen dominates over the displacement current or the displacement current dominates over conduction current" forms the basis for distinction between a conductor and a dielectric. For any medium the ratio of the conduction current density (J) and the displacement current density $J_D = \frac{\partial D}{\partial t}$ is given by $\frac{J}{J_D} = \frac{J}{\frac{\partial D}{\partial t}} = \frac{\sigma}{\omega\varepsilon}$ which is dimensionless

If $\frac{J}{\frac{\partial D}{\partial t}} = \frac{\sigma}{\omega\varepsilon} \geq 100$ then the conduction current dominates and the medium is conductor, however

If $\frac{\frac{\partial D}{\partial t}}{J} = \frac{\omega\varepsilon}{\sigma} \geq 100$ then the displacement current dominates and the medium is a dielectric. In between the medium is neither a conductor nor a dielectric rather it behaves as a semiconductor. For copper $\sigma = 5.8 x 10^7 \Omega^{-1} m^{-1}$ $\varepsilon = \varepsilon_0 = 8.86 x 10^{-12}$ F/m &

$\frac{\sigma}{\omega\varepsilon} = \frac{5.8 \text{ X } 10^7}{2\pi\upsilon \text{ X } 8.86\ 10^{-12}} = \frac{1.0424 \text{ X } 10^{18}}{\upsilon}$ for Ultra Violet radiation $\upsilon = 10^{16}$hz, so the specimen is a conductor for UV. For X-rays, $\upsilon = 10^{20}$hz, the ratio $\frac{\omega\varepsilon}{\sigma} \geq 100$ Hence the specimen is a dielectric for X-rays.

B4 Both F P Etalon and L G Plate are multiple beam interferometer. There are two optically plane parallel glass plates separated by distance t of space of refractive index μ in etalon while there is a single optically plane glass or quartz plate of uniform thickness t and refractive index μ in L G plate. In an etalon, the condition of interference is $2\mu t \cos\theta = n\lambda$ where θ is angle of incidence. The locus of constant θ being circle, the fringes are circular and are known as the fringes of equal inclination i,e the Haidinger fringes.

In L G Plate, the condition of interference is $2\mu t \cos r = n\lambda$ where r is the angle of refraction. The locus of constant r being a line, the fringes are parallel straight lines. With the increase of path difference the angle of emergence decreases resulting in higher order which says that higher order fringes lie higher in the field of view. The separation between the fringes is inversely proportional to the thickness of the plate. Unlike an etalon, the space between the two reflecting surfaces is dispersive medium. Also the Resolution in LG is inferior to that of a corresponding FP.

B5 Ultraviolet rays, coming from Sun, are injurious and harmful to human life (living bodies) Such rays are cancerous causing mottling in green leaves and make them lose chlorophyll. These are absorbed or reflected by small ozone layer present on the top of stratosphere and are not allowed to come to earth. In the absence of such a layer the chances of life on earth would have been remote.

B6 The formula for convex lens is $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ and that for a zone plate is $\frac{1}{a} + \frac{1}{b} = \frac{n\lambda}{r_n^2}$

The radius of n^{th} Zone of a zone plate is $r_n \propto \sqrt{n}$ and the two parameters a and b are analogous to u and v respectively. On changing the distance of the focus from zone plate, the radius of respective zones change but still satisfying the above focusing condition. Thus a zone plate exhibits multiple foci.

B7 The eigen values of Hermition operator are all real. Let ψ_1 and ψ_2 be the two eigen functions of Hermitian operator H for two different eigen values λ_1 and λ_2 such that

$H\psi_1 = \lambda_1\psi_1$ .... (1) & $H\psi_2 = \lambda_2\psi_2$ .... (2)

Pre multiplying eqn (1) by ψ_2^* and integrating we get $\int\psi_2^* H\psi_1 d\tau = \lambda_1\int\psi_2^*\psi_1 d\tau$ .... (3)

now Pre multiplying eqn (2) by ψ_1^*, and then taking complex conjugate we get $(\psi_1^* H\psi_2)^\dagger = (\lambda_2\psi_1^*\psi_2)^\dagger$ which yields

$\int\psi_2^* H\psi_1 d\tau = \lambda_2\int\psi_2^*\psi_1 d\tau$ .... (4)

[since $H^\dagger = H$ for Hermitian operator]

subtracting equation (3) from (4) one gets

$0 = \lambda_2\int\psi_2^*\psi_1 d\tau - \lambda_1\int\psi_2^*\psi_1 d\tau$ or $(\lambda_1 - \lambda_2)\int\psi_2^*\psi_1 d\tau = 0$

Further Since $\lambda_1 \neq \lambda_2$ one gets $\int\psi_2^*\psi_1 d\tau = 0$ showing that ψ_1 and ψ_2 are orthogonal wave functions.

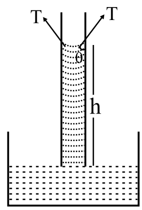
B8 Tangents drawn from an exterior point to a sphere include the surface which is less than a hemisphere. This is why, at least three satellites are required for worldwide transmission of a TV channel. These satellites really serve as reflectors of TV signal. In case of radio communication, a natural reflector is provided by ionosphere through which the frequency modulated high frequency waves which are used in TV transmission are penetrated and are not reflected. In other words Ionosphere is nearly transparent for TV waves

B9 Certain molecules when illuminated by radiations of definite frequency (one of its absorption frequencies), are raised to excited electronic states and revert back to their initial state, after a defined life time $t \approx 10^{-8}$ s, with the emission of radiations of discrete frequencies equal to or smaller than the incident frequency. This phenomenon is known as luminescence. The emission may vanish almost immediately after the removal of exciting radiations or it may persist for an appreciable time. The former is known as fluorescence and the later as phosphorescence. Their combined name is luminescence.

B10 In a semiconductor, more and more covalent bonds break with the rise in temperature as a result the number of carriers increases while the mobility decreases because of frequent collisions. The impact of increase in number of carriers dominates over the decrease in mobility, consequently the conductivity increases. Beyond a certain temperature there is no increase in the carrier concentration which rather reaches the saturation. The decrease in mobility with rise in temperature then decreases the electrical conductivity of the semiconductor.

Part B -II

P1(a) In a vertical capillary of radius (R) the liquid rises due to surface tension (T) to a height $h = \dfrac{2T\cos\theta}{\rho g R}$



The force acting vertically upward on the liquid column is $F = 2\pi R\, T\cos\theta$. Therefore the work done is $W = Fh = 2\pi RT\cos\theta \times \dfrac{2T\cos\theta}{\rho g R}$

or $W = \dfrac{4\pi T^2\cos^2\theta}{\rho g}$ The change in the potential energy is $U = \pi R^2 h\rho \times g \times \dfrac{h}{2}$ or

$$U = \left[\frac{\pi R^2 \rho g}{2}\right] \times \left[\frac{2T\cos\theta}{\rho g R}\right]^2 = \frac{2\pi T^2\cos^2\theta}{\rho g}$$

By energy considerations, one can write work done by Surface Tension – increase in potential energy = heat produced (dQ) . Mathematically $W - U = dQ$. Thus

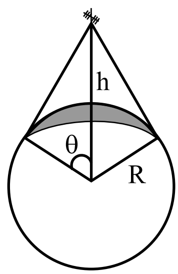
$$dQ = \frac{4\pi T^2\cos^2\theta}{\rho g} - \frac{2\pi T^2\cos^2\theta}{\rho g} = \frac{2\pi T^2\cos^2\theta}{\rho g}$$

In case of perfect wetting of glass, the angle of contact $\theta = 0°$ or $\cos\theta = 1$. Then $dQ = \dfrac{2\pi T^2}{\rho g}$

(b) The area on the earth surface, to receive the direct signal from TV transmitter is subtending a solid angle $d\Omega = \dfrac{\Delta S}{R^2} \Rightarrow \Delta S = R^2 d\Omega$

$$\Rightarrow \Delta S = \int_0^\theta R^2 d\Omega$$

$$= R^2 2\pi\,(1 - \cos\theta)$$



The height of tower is h = 100 m, than

$$\Delta S = R^2 2\pi \left(1 - \frac{R}{R+h}\right)$$

$$= \frac{2\pi h R^2}{R+h}$$

Therefore the population covered $= \Delta S \times N$

$$= \frac{2\pi h R^2}{R+h} \times 10^3 = \frac{2\pi 100 \times (6.37)^2}{(6.37 \times 10^6 + 100)} \times 10^{19} = 4.0 \times 10^6$$

Thus the population covered by the TV transmission is four Million.

P2 Mass of the Cylinder + two cones

$M = \pi R^2 h\rho + 2\pi R^2 \frac{h}{3}\rho = \frac{5}{3}\pi R^2 h\rho$

Now moment of Inertia of the cylinder is $I_{cyl} = \frac{1}{2}\pi R^2 h\rho R^2$

of two cones is $I_{cone} = 2 \times \frac{3}{10}\{\pi R^2 \frac{h}{3}\rho\}R^2$

Total $I = I_{cyl} + I_{cone}$

$I = \frac{1}{2}\pi R^2 h\rho R^2 + 2 \times \frac{3}{10}\{\pi R^2 \frac{h}{3}\rho\}R^2 = \mathbf{\frac{21}{50}MR^2}$

P3(a) The energy carried by an EM wave is the Poynting vector(s)

$S = \mathbf{E} \times \mathbf{H} = EH = \frac{2 \times 4.2}{60 \times 10^{-4}} = 1400 J/m^2$

Also

$\frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 4\pi\sqrt{\frac{\mu_0}{4\pi \times 4\pi \times \varepsilon_0}} = 4\pi \times 30 = 377\Omega$

therefore $E^2 = 1400 \times 377 \Rightarrow E = 726.5$ V/m

and $726.5 \times H = 1400 J/m^2 \Rightarrow H = 1.93 \frac{A-turn}{m}$

These expressions yield rms values. The amplitude of Electric and magnetic fields are $E_0 = E\sqrt{2} = 726.5 \times 1.414 = 1027.4$ V/m and $H_0 = H\sqrt{2} = 1.93 \times 1.414 = 2.73$ A-turn/m and $B_0 = \mu_0 H_0 = 4\pi \times 10^{-7} \times 2.73 = 3.43 \times 10^{-6}$ T

(b) The intensity of the concentrated beam is equal to the Poynting vector (s).

$I = \frac{200}{4 \times 10^{-4} \times 10^{-4}} = 5 \times 10^9$ watt/m^2

$S = \frac{\mathbf{E} \times \mathbf{B}}{2\mu_0} = \frac{E^2}{2c\mu_0} = 5 \times 10^9$ watt/m^2

$\Rightarrow E^2 = 2 \times 5 \times 10^9 \times 3 \times 10^8 \times 4\pi \times 10^{-7}$

$E = 1.94 \times 10^6$ V/m

Alternately $I = \frac{200}{4 \times 10^{-4} \times 10^{-4}} = 5 \times 10^9$ w/m^2

Further $U = \frac{I}{C} = \frac{5 \times 10^9}{3 \times 10^8} = \frac{50}{3}\frac{J}{m^3}$

and now $U = \frac{\varepsilon_0 E^2}{2} = \frac{50}{3}\frac{J}{m^3} \Rightarrow E = 1.94 \times 10^6$ V/m

P4 Just when the switch is closed (Refer figure in the question paper), the inductor is an open circuit hence the resistance R=2 Ω in parallel to the inductor will be effective while the capacitor will be short circuited (giving zero resistance) so the current is $i = \frac{2}{2+1} = \frac{2}{3}$ A

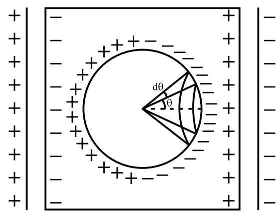
Long time after the capacitor gets charged and offers infinite resistance while the inductor gets short circuited hence the current is

$i = \frac{2}{6+1} = \frac{2}{7}$ A

When turned off the ammeter is not connected hence the current through it is zero. Even though the charged capacitor will discharge through 6 Ω resistor.

Ans: **(i) i = 2/3 amp (ii) i = 2/7 amp (iii) i = zero**

P5. A spherical cavity is considered in side a dielectric placed between the plates of a charged capacitor. The charge is so induced that one hemisphere has a positive charge and the other half has negative charge. The polarization vector P is defined as the induced charge per unit surface area.



As shown in the figure we consider an element on the surface of the cavity between angles θ and θ + dθ. The area of such an element on the surface is $dS = 2\pi R \sin\theta \times R d\theta$.

With the polarization vector **P**, the charge on this element is $dq = \mathbf{P.dS} = PdS\cos\theta$

$dq = P 2\pi R^2 \sin\theta \cos\theta \, d\theta$. The electric field produced by this charge at the centre of the cavity is $dE = \frac{1}{4\pi\varepsilon_0}\frac{2\pi R^2 P \sin\theta \cos\theta \, d\theta}{R^2}$

directed radially out ward. The components of E shall be $\sum dE \sin\theta = 0$ and $\sum dE \cos\theta = E$

Thus the resultant field (E) will be

$$E = 2\sum dE\cos\theta = 2\frac{1}{4\pi\varepsilon_0}\int_0^{\frac{\pi}{2}} 2\pi P \sin\theta \cos\theta \, d\theta \cos\theta$$

$$E = \frac{P}{\varepsilon_0}\int_0^{\frac{\pi}{2}} \cos^2\theta \sin\theta \, d\theta \Rightarrow E = \frac{P}{3\varepsilon_0}$$

P6. The de Broglie wavelength (λ) for a particle is $\lambda = \frac{h}{p} = \frac{h}{mv}$

It is known that relativistic KE = $mc^2 - m_0c^2$

Under the influence of an electric potential V, the kinetic energy is eV so $E = mc^2 = eV + m_0c^2$

$$E = mc^2 = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow E^2\left\{1-\frac{v^2}{c^2}\right\} = m_0^2c^4$$

or $E^2 - m^2v^2c^2 = E^2 - p^2c^2 = m_0^2c^4$

$\Rightarrow (eV + m_0c^2)^2 - p^2c^2 = m_0^2c^4$

or $p^2c^2 = e^2V^2 + 2eVm_0c^2$

$$\text{or } p = \sqrt{2m_0eV\left(1+\frac{eV}{2m_0c^2}\right)}$$

$$\text{Thus } \lambda = \frac{h}{\sqrt{2m_0eV\left(1+\frac{eV}{2m_0c^2}\right)}}$$

Hence proved.

P7. Photon energy incident per second is power $P = ncAh\upsilon = 100$ watt (given)

n being the number of photons per unit volume and υ is frequency of incident radiation. As the angular

momentum of photon is $\hbar = \left(\frac{h}{2\pi}\right)$, the angular momentum transfered per second by the incident photon flux is

$$\frac{\Delta J}{\Delta t} = ncA\hbar = ncA\left(\frac{h}{2\pi}\right) = \frac{P}{2\pi\upsilon} = \frac{P\lambda}{2\pi c}$$

$$= \frac{100 \times 500 \times 10^{-9}}{2 \times 3.14 \times 3 \times 10^{8}} = 2.65 \times 10^{-14} \text{ kg m}^2/\text{S}^2$$

If the total time elapsed for which, the momentum transfer takes place is t then using

$$\frac{\Delta J}{\Delta t} = I\frac{d\omega}{dt} = \frac{P\lambda}{2\pi c} \Rightarrow \int_0^t dt = \frac{2\pi cI}{P\lambda}\int_0^{2\pi} d\omega$$

$$\text{There by } t = \frac{1\times10^{-5} \times (2.5 \times 10^{-3})^2\, 2\pi \times 1}{2 \times 2.65 \times 10^{-14}}$$

T = 7410 S = 2 hour and 3.5 min

P8. According to Dulong and Petit's law, the specific heat of solids is constant at all values of temperature and is $C_V = 3R$. Measurements at low temperature have revealed that the specific heat decreases at low temperature as T^3 which is consistent with Debye model where one considers the atoms as strongly coupled harmonic oscillators Debye frequency (υ_D) is expressed as $\upsilon_D^3 = \frac{9N}{4\pi V}\left\{\frac{1}{v_t^3} + \frac{2}{v_t^3}\right\}^{-1}$

N being the total number of atoms present in volume V of the substance.

Debye temperature is defined as $\theta_D = \frac{h\upsilon_D}{K}$

Further considering the longitudinal velocity v_l = transverse velocity v_t = mean velocity of sound v_s, Debye frequency can be expressed as

$$\upsilon_D = v_S\left\{\frac{3N}{4\pi V}\right\}^{\frac{1}{3}} \quad \text{and}$$

Debye temperature as $\theta_D = \frac{hv_S}{K}\left\{\frac{3N}{4\pi V}\right\}^{\frac{1}{3}}$

For one mole of the substance it may further be written as $\theta_D = \frac{hv_S}{K}\left\{\frac{3\rho N_A}{4\pi M}\right\}^{\frac{1}{3}}$ Comparing these results for Gold (Au) and Copper (Cu)

$$\frac{(\theta_D)_{Au}}{(\theta_D)_{Cu}} = \frac{(v_s)_{Au}}{(v_s)_{Cu}}\left\{\left(\frac{\rho}{M}\right)_{Au}\left(\frac{M}{\rho}\right)_{Cu}\right\}^{\frac{1}{3}}$$

$$= \left(\frac{2100}{3800}\right)\left(\frac{19.3 \times 63.54}{8.96 \times 197.0}\right)^{\frac{1}{3}} = 0.5526 \times (0.695)^{\frac{1}{3}}$$

$= 0.5526 \times 0.88568 = 0.4894$

Therefore $(\theta_D)_{gold} = 0.4894 \times 348 = 170$K

P9 When charged by pressing the Morse key, the capacitor acquires the charge $q_0 = CV$ .... (1)

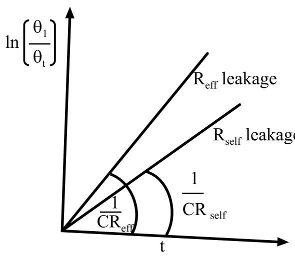
Discharging through BG, one notes the first throw θ_1 such that $q_0 = K\theta_1\left(1-\frac{\lambda}{2}\right)$ .... (2)

The charge on the capacitor after discharging through a high resistance R for a time t is

$$q = q_0 e^{-\frac{t}{CR}} = K\theta_t\left[1 - \frac{\lambda}{2}\right] \quad .... (3)$$

From equations (2) & (3) $\frac{q_0}{q} = e^{\frac{t}{CR}} = \frac{\theta_1}{\theta_t}$

Or $\frac{t}{CR} = \ln\left(\frac{\theta_1}{\theta_t}\right)$ This shows that the plot of $\ln\left(\frac{\theta_1}{\theta_t}\right)$ v/s t is a straight line through origin with its slope = $\frac{1}{CR}$ and is shown in figure below



Further since the self leakage resistance is much greater than the resistance R, the slope will be less in the second case

where $\ln\left(\frac{\theta_1}{\theta_{ts}}\right)$ v/s t is plotted

The resistance can be expressed as

$$R_{eff} = \frac{t}{C\ln\left(\frac{\theta_1}{\theta_t}\right)} \quad \text{and} \quad R_s = \frac{t}{C\ln\left(\frac{\theta_1}{\theta_{ts}}\right)}$$

The effective resistance shall be obtained by an addition of these two parallel resistances as

$$\frac{1}{R_{eff}} = \frac{1}{R} + \frac{1}{R_s} \quad \text{or} \quad \frac{1}{R} = \frac{1}{R_{eff}} - \frac{1}{R_s}$$

$$\frac{1}{R} = \frac{C\ln\left(\frac{\theta_1}{\theta_t}\right)}{t} - \frac{C\ln\left(\frac{\theta_1}{\theta_{ts}}\right)}{t}$$

$$\frac{1}{R} = C\left[\tan\frac{\pi}{4} - \tan\frac{\pi}{6}\right] = 0.1 \times 10^{-6}\left[1 - \frac{1}{\sqrt{3}}\right]$$

$\Rightarrow R = 23.66\ M\Omega$

P10(a)The number of electron hole pairs is

$n = N_0 e^{-\frac{E_g}{kT}}$ Thereby the carriers will be

at 27°C, $n_{27} = N_0 e^{-\frac{E_g}{k(27+273)}}$ and

at 47°C, $n_{47} = N_0 e^{-\frac{E_g}{k(47+273)}}$

Thus $\frac{n_{47}}{n_{27}} = e^{\frac{E_g}{300k} - \frac{E_g}{320k}} = e^{\frac{E_g}{k}\left[\frac{1}{300} - \frac{1}{320}\right]}$

$\frac{n_{47}}{n_{27}} = e^{\frac{0.7 \times 1.6\ 10^{-19}}{1.38\ 10^{-23}}\left[\frac{1}{300} - \frac{1}{320}\right]} = 5.42$ means

the number of carriers become 5.42 times. The percent change in electron hole pairs is

$$\frac{n_{47} - n_{27}}{n_{27}} \times 100\% = [5.42 - 1] \times 100 = 442\%.$$

(b) The loop gain of an amplifier is just the power gain and may be expressed as

$$A_l = 20 \log \frac{V_{out}}{V_{in}} = 60 \text{ (given)}$$

Thus $V_{out} = 10^3 V_{in} = 1000\, V_{in}$

Due to the aging effect, the change in gain is

20% there by $\frac{dA}{A} \times 100 = 20$ or $\frac{dA}{A} = 0.2$.

If the closed loop gain (the feed back) is A_f

then $\frac{dA_f}{A_f} \times 100 = 2$ (required) or $\frac{dA_f}{A_f} = 0.02$

Furher $\frac{dA_f}{A_f} = \frac{1}{1 - A\beta} \times \frac{dA}{A}$

Therefore to achieve a 2% gain we write

$$0.02 = \frac{1}{1 - 1000\beta} \times 0.2 \Longrightarrow \beta = -0.009$$

Substituting value of β

$$A_f = \frac{A}{1 - A\beta} = \frac{1000}{1 - 1000(-0.0009)} = 100$$

Thus the required closed loop gain (feed back) is A_f=100.